

Answer **all** questions in the spaces provided.

- 1 Write $30 : 12$ in the form $n : 1$

[1 mark]

$$\begin{array}{c} \div 12 \quad \downarrow \quad 30 : 12 \quad \downarrow \quad \div 12 \\ \quad \quad 2.5 : 1 \quad \quad \end{array}$$

Answer 2.5 : 1

- 2 Four consecutive triangular numbers are

$$\begin{array}{ccccccc} & & +4 & & +6 & & \\ & \nearrow & & \nearrow & & \nearrow & \\ 6 & & 10 & & 15 & & 21 \\ & \nwarrow & & \nwarrow & & \nwarrow & \\ & & +5 & & +7 & & \end{array}$$

Write down the next triangular number.

[1 mark]

$$21 + 7 = 28$$

Answer 28

Answer **all** questions in the spaces provided.

Do not write
outside the
box

- 1** Write $28 : 8$ in the form $n : 1$

[1 mark]

Answer _____ : 1

- 2** Four consecutive terms from the Fibonacci sequence are 3 5 8 13
Write down the next term.

[1 mark]

Answer _____

Turn over ►

- 3 Write down the reciprocal of $\frac{4}{7}$ [1 mark]

Answer $\frac{7}{4}$

- 4 The price of a toy increases by 12.5% to £19.53
Work out the **original** price of the toy.

[2 marks]

$$\begin{array}{r|l} 112.5\% & \text{£}19.53 \\ \hline 100\% & \text{£}17.36 \end{array} \quad \begin{array}{l} \nearrow \times 1.125 \\ \searrow \div 1.125 \end{array}$$

Answer £ 17.36

- 3 Write down the reciprocal of $\frac{5}{8}$ [1 mark]

Answer _____

- 4 The price of a necklace increases by 37.5% to £38.17
Work out the **original** price of the necklace. [2 marks]

Answer £ _____

5

Jess saves 2p, 5p and 10p coins.

She has

- 45 10p coins
- 8 times as many 2p coins as **10p coins**
- £17.70 in total.

Work out total **value** of 2p coins : total **value** of 5p coins

Give your answer in its simplest form.

[4 marks]

$$45 \times 10p = 450p = £4.50$$

$$8 \times 45 = 360 \quad 360 \times 2p = 720p = £7.20$$

$$4.50 + 7.20 = 11.70$$

$$17.70 - 11.70 = £6.00 \text{ in } 5p \text{ coins}$$

$$£7.20 : £6.00$$

$$\div 10 \left(\begin{array}{l} 720 : 600 \end{array} \right) \div 10$$

$$\div 12 \left(\begin{array}{l} 72 : 60 \\ 6 : 5 \end{array} \right) \div 12$$

Answer 6 : 5

She has

- Work out total **value** of 2p coins : total **value** of 5p coins

[4 marks]

Answer _____ :

- 6 (a) Part of a regular polygon is shown.



Not drawn
accurately

Assume that the polygon is an octagon.

Work out the size of an **exterior** angle.

[2 marks]

Exterior angles sum to 360°

$$\frac{360^\circ}{8} = 45^\circ$$

Answer 45 °

- 6 (b) In fact, the polygon has **more** sides than an octagon.

$$\frac{360}{9} = 40^\circ$$

What does this mean about the size of an exterior angle?

Tick **one** box.

$$\frac{360}{10} = 36^\circ$$

[1 mark]

☐

It is more than the answer to part (a)

☐

It is the same as the answer to part (a)

☒

It is less than the answer to part (a)

☐

It could be any of the above

- 6 (a) Part of a regular polygon is shown.



Not drawn
accurately

Assume that the polygon is a hexagon.

Work out the size of an **exterior** angle.

[2 marks]

Answer _____ °

- 6 (b) In fact, the polygon has **more** sides than a hexagon.

What does this mean about the size of an exterior angle?

Tick **one** box.

[1 mark]

☐

It is more than the answer to part (a)

☐

It is the same as the answer to part (a)

☐

It is less than the answer to part (a)

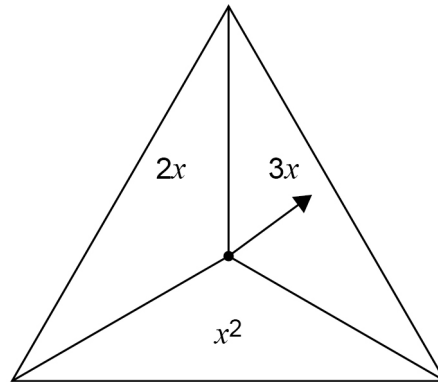
☐

It could be any of the above

7

In a game,

- an ordinary fair six-sided dice is rolled
- the fair spinner shown is spun.



The score is the dice number **substituted** into the spinner expression.

7 (a) Complete the table to show all of the possible scores.

[2 marks]

	1	2	3	4	5	6
$2x$	2	4	6	8	10 ✓	12 ✓
$3x$	3	6	9	12 ✓	15 ✓	18 ✓
x^2	1	4	9	16 ✓	25 ✓	36 ✓

- 7 (b) A player wins the game if their score is 10 or more.

Work out the probability that they win the game.

[1 mark]

8 scores win

Answer $\frac{8}{18} = \frac{4}{9}$

- 7 (c) The game is played 711 times.

Estimate the number of games that are won.

[2 marks]

$\frac{4}{9} \times 711 = 316$

Answer 316

8 $(a - 3)x^2 + 2b \equiv 5x^2 + 12$

Work out the values of a and b .

[2 marks]

$a - 3 = 5$
 $+3 \quad +3$
 $a = 8$

$2b = 12$
 $\div 2 \quad \div 2$
 $b = 6$

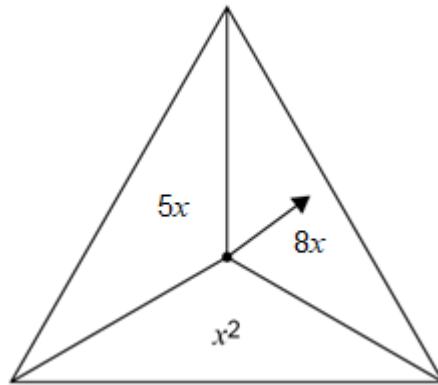
$a = 8 \quad b = 6$

Turn over ►

7

In a game,

- an ordinary fair six-sided dice is rolled
- the fair spinner shown is spun.



The score is the number on the dice **substituted** into the spinner expression.

7 (a) Complete the table to show all of the possible scores.

[2 marks]

	1	2	3	4	5	6
$5x$						30
$8x$		16				
x^2				16		

- 7 (b)** A player wins the game if their score is 30 or more.

Work out the probability that they win the game.

[1 mark]

Answer _____

- 7 (c)** The game is played 756 times.

Estimate the number of games that are won.

[2 marks]

Answer _____

8 $(a - 5)x^2 + 4b \equiv 3x^2 + 20$

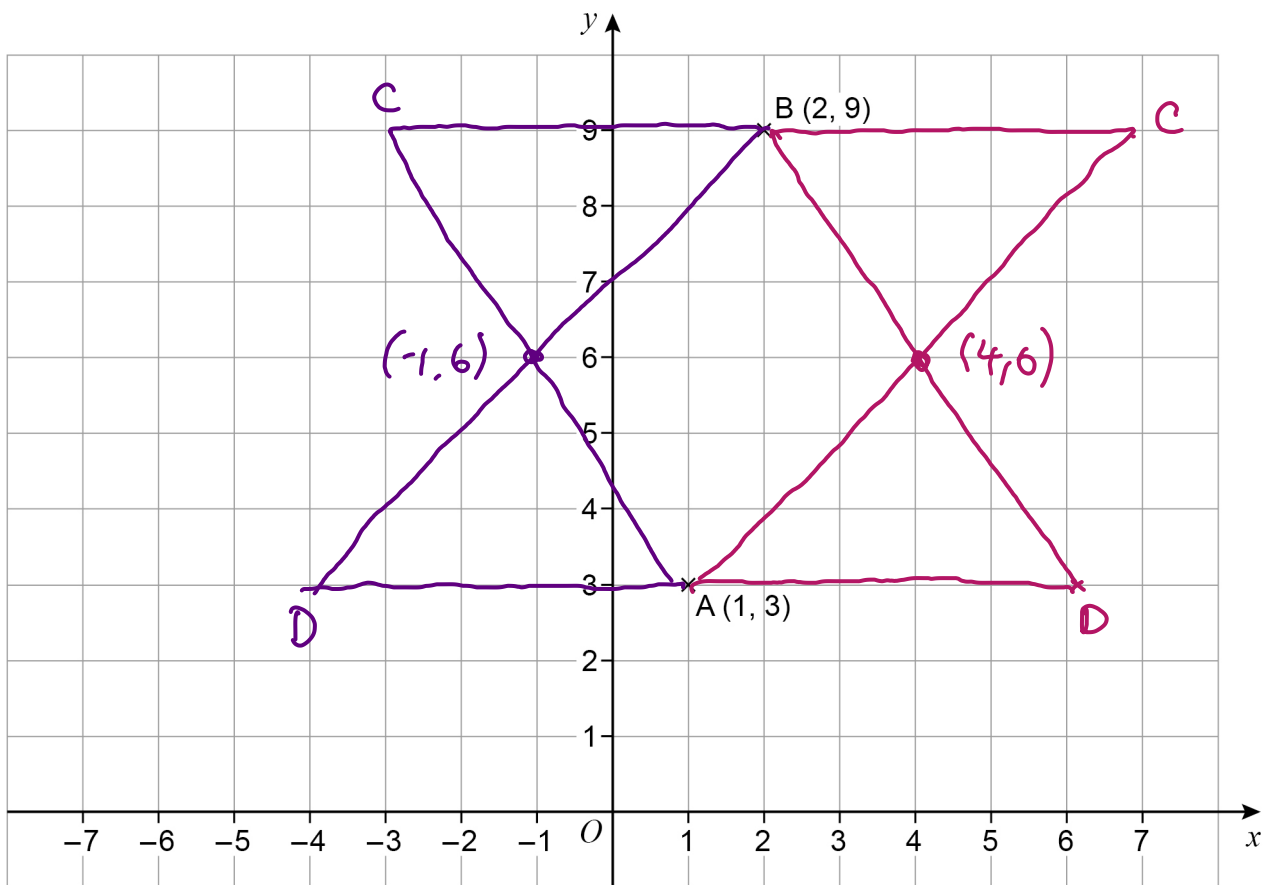
Work out the values of a and b .

[2 marks]

$a =$ _____ $b =$ _____

9

A (1, 3) and B (2, 9) are points on a centimetre grid.



ABCD is a parallelogram.

AD and BC are **horizontal** and each has length 5 cm

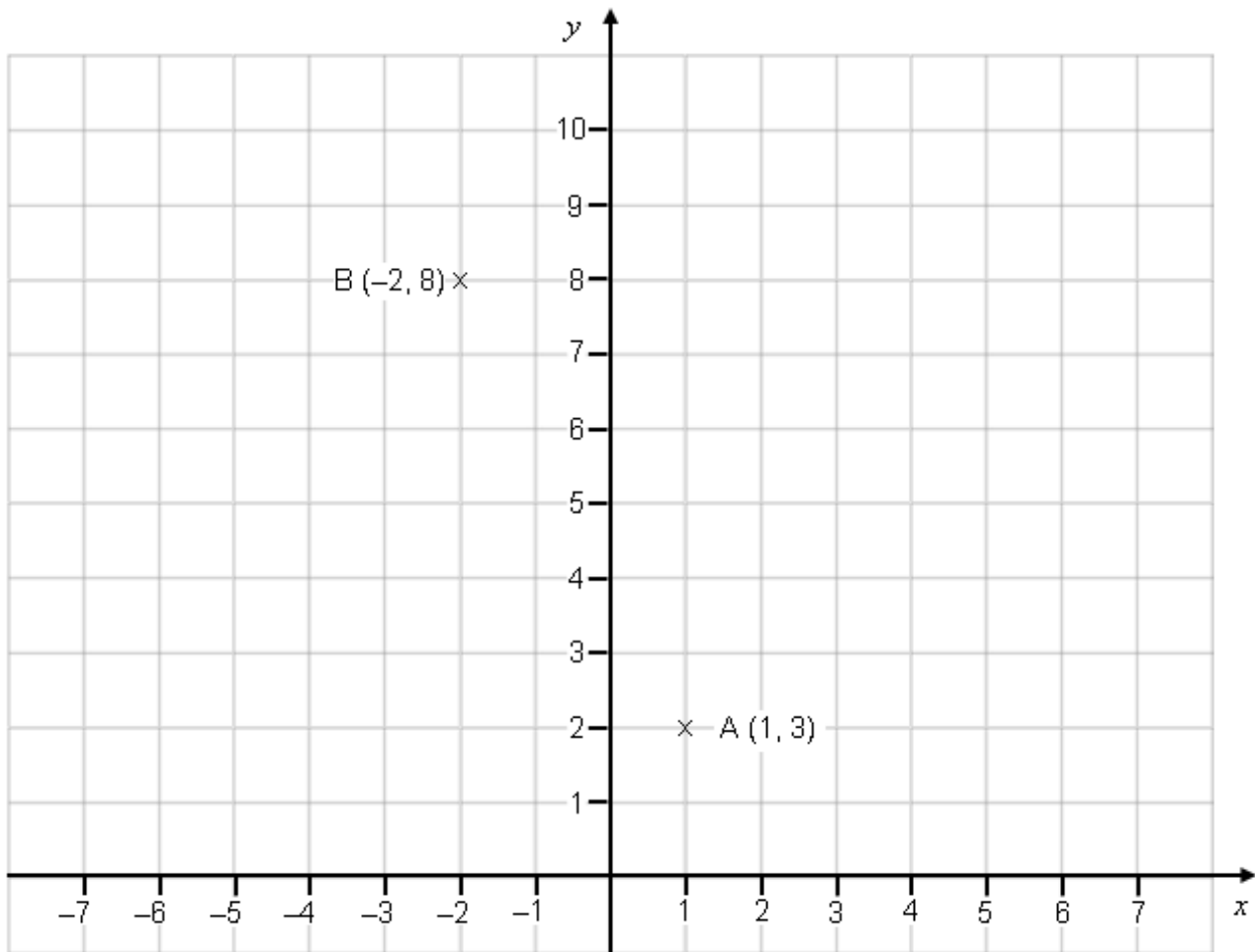
The diagonals of ABCD cross at E.

Work out the **two** possible pairs of coordinates of E.**[4 marks]**

Answer (4 , 6) and (-1 , 6)

9

A (1, 2) and B (-2, 8) are points on a centimetre grid.



ABCD is a parallelogram.

AD and BC are **horizontal** and each has length 5 cm

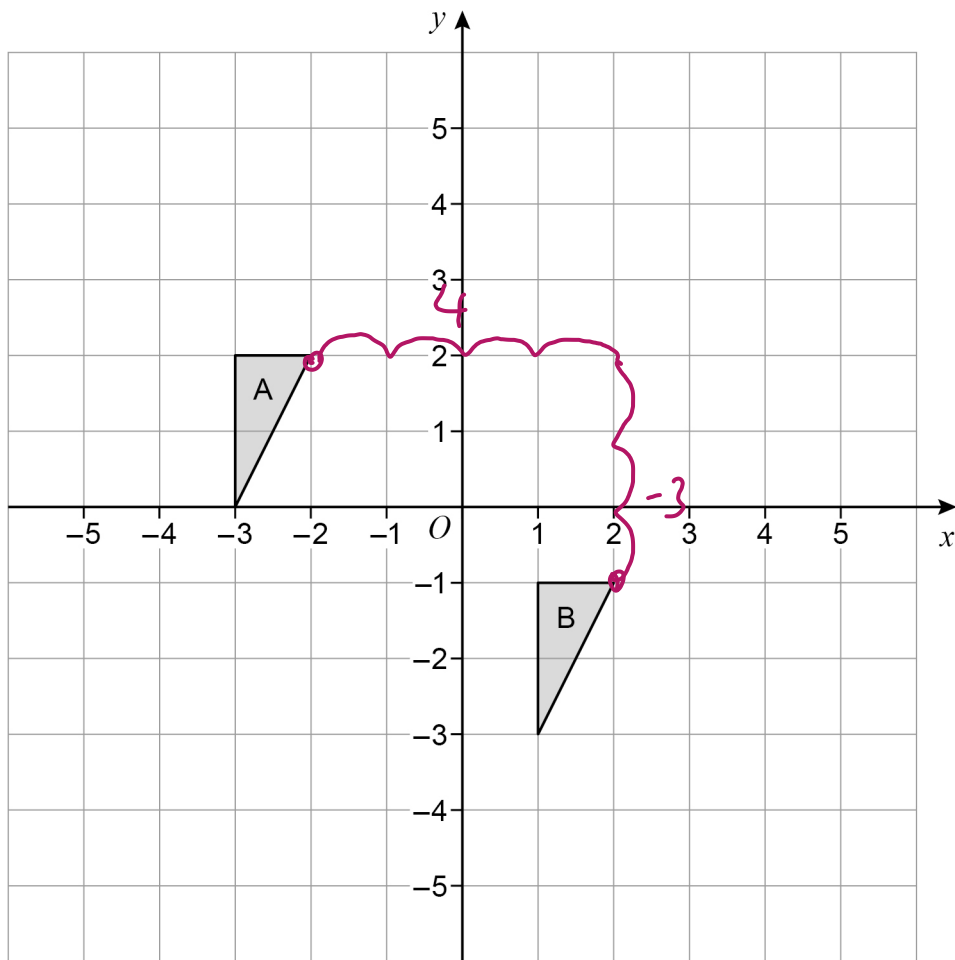
The diagonals of ABCD cross at E.

Work out the **two** possible pairs of coordinates of E.**[4 marks]**

Answer (_____ , _____) and (_____ , _____)

10

Write down the translation vector that maps shape A onto shape B.

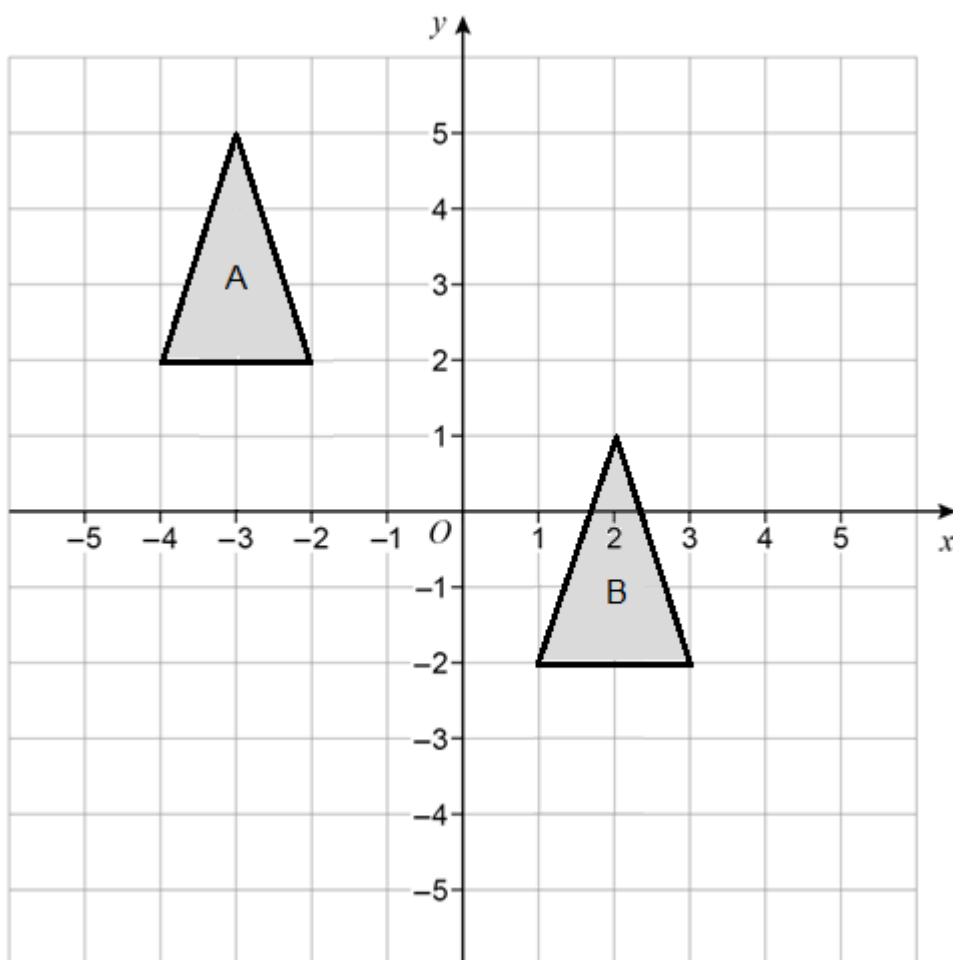
[2 marks]

Answer _____

 $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

10

Write down the translation vector that maps shape A onto shape B.

[2 marks]

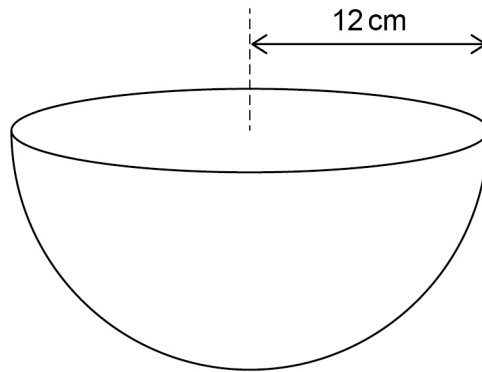
Answer _____

Turn over ►

11

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

A bowl is a hemisphere with radius 12 cm



Water is poured into the bowl
at a rate of 325 cm^3 per second
for 8 seconds.

Does the water fill **more than** 70% of the bowl?

You **must** show your working.

[4 marks]

$$\text{Water} : 325 \times 8 = 2600 \text{ cm}^3$$

$$\text{Volume of hemisphere} : \frac{1}{2} \text{ of } \frac{4}{3} \pi r^3 = \frac{2}{3} \pi r^3$$

$$\frac{2}{3} \times \pi \times 12^3 = 3619.114737$$

$$70\% \text{ of volume} : 0.7 \times 3619.114737 = 2533.380316$$

Yes.

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

A diagram of a hemispherical bowl. A vertical dashed line extends from the center of the circular rim to the bottom of the bowl. A horizontal double-headed arrow indicates the radius from this dashed line to the rim, labeled "9 cm".

Does the water fill **more than** 80% of the bowl?
You **must** show your working.

[4 marks]

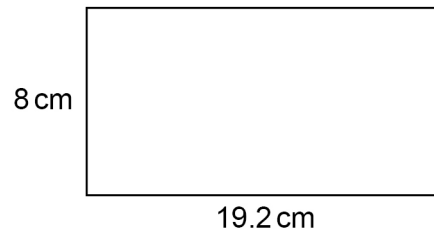
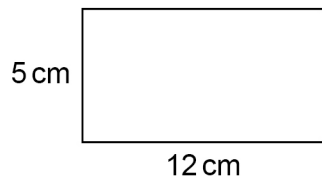
4

Turn over ►

- 12 Show that these two rectangles are similar.

[2 marks]

Not drawn
accurately



$$8 \div 5 = 1.6$$

$$19.2 \div 12 = 1.6$$

- 13 A factory packs x boxes of teabags per hour.
Each box contains 80 teabags.

Show that the factory packs $\frac{4x}{3}$ teabags per minute.

[2 marks]

$$80 \times x = 80x \text{ teabags per hour}$$

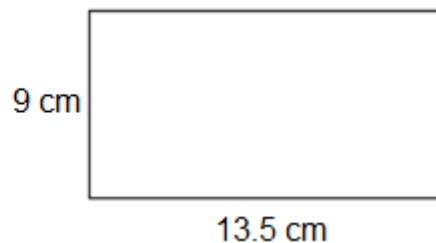
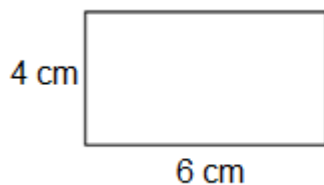
$$\downarrow \div 60$$

$$\frac{80x}{60} \text{ teabags per minute} = \frac{4x}{3}$$

- 12** Show that these two rectangles are similar.

[2 marks]

Not drawn
accurately



- 13** A factory packs x boxes of plasters per hour.
Each box contains 100 plasters.

Show that the factory packs $\frac{5x}{3}$ plasters per minute.

[2 marks]

Turn over ►

14

A company has 123 employees.

Information about their hourly rates of pay is shown in the table.

Hourly rate, £ p	Number of employees
$10 \leq p < 14$	66
$14 \leq p < 20$	32 $(\frac{1}{2})A$
$20 \leq p < 40$	15 A
$40 \leq p < 100$	10 A
Total = 123	

The owner of the company uses the data to make two statements.

Statement A

“Over 30% of employees have an hourly rate that is more than £17”

Statement B

“The average hourly rate of pay is more than £20”

14 (a) Show working that supports **Statement A**.

[3 marks]

$$10 + 15 + 16 = 41$$

$$\frac{41}{123} \times 100 = 33.3\%$$

14 (b) Why might **Statement A** not be true?

[1 mark]

The 32 employees in the $14 \leq p < 20$ interval might not be evenly distributed. If more earn £14, this would make statement A false.

14 (c) Work out an estimate of the mean to support **Statement B**.

[3 marks]

mp f

$$10 \times 66 = 660$$

$$17 \times 32 = 544$$

$$30 \times 15 = 450$$

$$70 \times 10 = 700$$

$$\underline{2354}$$

$$\frac{2354}{123} = 19.13821138$$

$$£19.14 > £18.$$

14 (d) Why is the mean **not** the best average to represent the data?

[1 mark]

Extreme values (hourly pay of £100) dramatically increase the mean

Turn over ►

14

A company has 113 employees.

Information about their hourly rates of pay is shown in the table.

Hourly rate, £ p	Number of employees
$8 \leq p < 12$	56
$12 \leq p < 20$	28
$20 \leq p < 40$	17
$40 \leq p < 60$	12
	Total = 113

The owner of the company uses the data to make two statements.

Statement A

“Over 35% of employees have an hourly rate that is more than £16”

Statement B

“The average hourly rate of pay is more than £18”

14 (a) Show working that supports **Statement A**.

[3 marks]

14 (b) Why might **Statement A** not be true?

[1 mark]

14 (c) Work out an estimate of the mean to support **Statement B**.

[3 marks]

14 (d) Why is the mean **not** the best average to represent the data?

[1 mark]

- 15 Expand $(x^2 - 9xy)(2x + 5y)$ [2 marks]

$$\begin{array}{r|l} x & x^2 - 9xy \\ \hline 2x & 2x^3 - 18x^2y \\ +5y & 5x^2y - 45xy^2 \end{array} \quad 2x^3 - 13x^2y - 45xy^2$$

Answer $2x^3 - 13x^2y - 45xy^2$

- 16 Line A
has equation $y = ax - 1$
passes through the point $(7, 13)$

Line B has equation $5y - 3x = 4$

Show that line A has a greater gradient than line B.

[3 marks]

$$y = ax - 1 \quad (7, 13) \rightarrow 13 = 7a - 1$$

$$+1 \quad +1$$

$$14 = 7a$$

$$\div 7 \quad \div 7$$

$$2 = a$$

$$y = 2x - 1$$

$$m_A = 2$$

$$5y - 3x = 4$$

$$+3x \quad +3x$$

$$5y = 3x + 4$$

$$\div 5 \quad \div 5$$

$$y = \frac{3}{5}x + \frac{4}{5}$$

$$m_B = \frac{3}{5}$$

$$2 > \frac{3}{5}$$

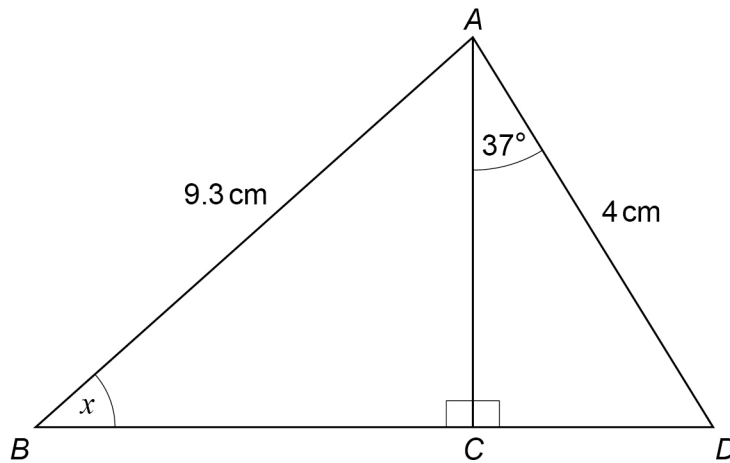
[2 marks]

16 Line A
has equation $y = ax - 5$
passes through the point (9, 22)

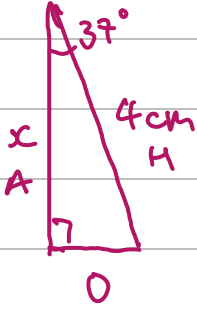
Show that line A has a greater gradient than line B.

[3 marks]

17

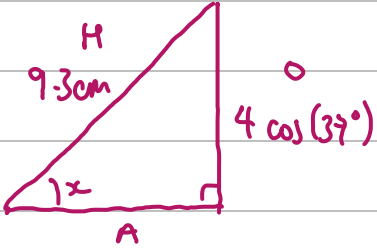
Not drawn
accuratelyWork out the size of angle x .

[4 marks]


 $\cos \theta = \frac{A}{H}$

$\cos 37^\circ = \frac{x}{4}$

$(\cos 37^\circ) \times 4 = x = 4 \cos(37)^\circ$


 $\sin \theta = \frac{O}{H}$

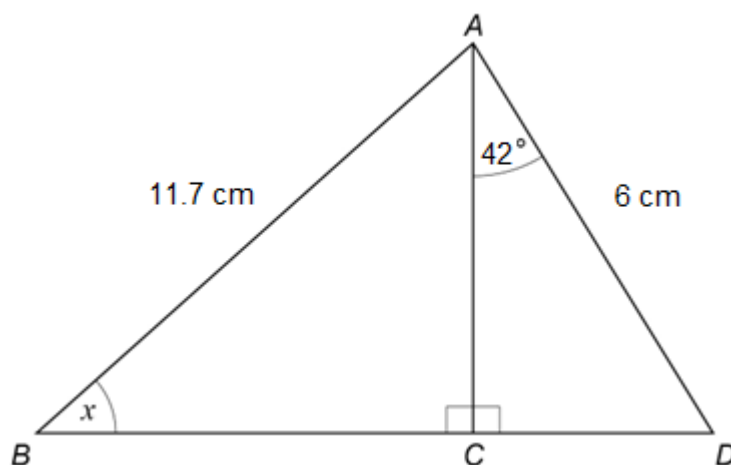
$\sin x^\circ = \frac{4 \cos(37)}{9.3}$

$x^\circ = \sin^{-1}\left(\frac{4 \cos(37)}{9.3}\right) = 20.09020484$

≈ 20.1

$x = 20.1^\circ$

Not drawn
accurately



[4 marks]

$x =$ _____

4

Turn over ►

18

Rearrange $y = \frac{x+8}{x}$ to make x the subject.

[3 marks]

$$y = \frac{x+8}{x}$$

$$xy = x+8$$

$$xy - x = 8$$

$$x(y-1) = 8$$

$$\div (y-1) \quad \div (y-1)$$

$$x = \frac{8}{y-1}$$

Answer $x = \frac{8}{y-1}$

18 Rearrange $z = \frac{xy + 4}{x}$ to make x the subject.

[3 marks]

Answer _____

19

Here are the first four terms of a quadratic sequence.

3 20 47 84

Work out an expression for the n th term of the sequence.**[4 marks]**

$$\begin{array}{ccccccc}
 3 & & 20 & & 47 & & 84 \\
 & \nearrow +17 & & \nearrow +27 & & \nearrow +37 & \\
 & & & \nearrow +10 & & \nearrow +10 & \\
 & & & & & & 10 \div 2 = 5
 \end{array}$$

$$\begin{array}{ccccccc}
 (5n^2) & 5 & 20 & 45 & 80 & & \\
 \underline{-2} & \underline{0} & \underline{+2} & \underline{+4} & & (2n-4) & \\
 3 & 20 & 47 & 84 & & &
 \end{array}$$

$$(5n^2) + (2n - 4)$$

Answer $5n^2 + 2n - 4$

Here are the first four terms of a quadratic sequence.

Work out an expression for the n th term of the sequence.

[4 marks]

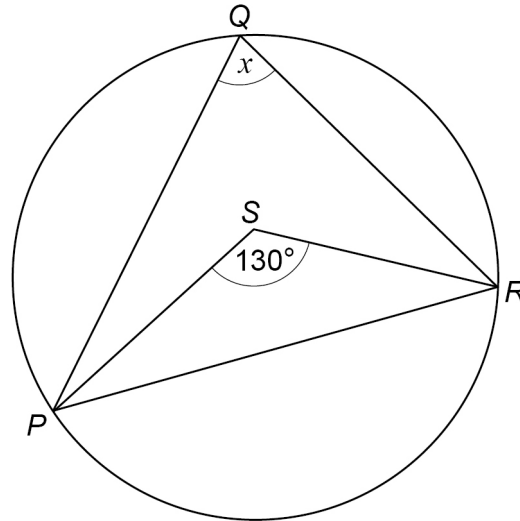
Answer

4

Turn over ►

- 20 (a)** P , Q and R are points on a circle.
 S is a point inside triangle PQR .

Not drawn
accurately



Assume that S is the centre of the circle.

Work out the size of angle x .

[1 mark]

$$130 \div 2 = 65$$

$$x = 65^\circ$$

- 20 (b)** In fact, the centre of the circle is on PS but **not** at S .

What does this mean about the size of angle x ?

Tick **one** box.

[1 mark]

☐

It is the same as the answer to part (a)

☒

It is greater than the answer to part (a)

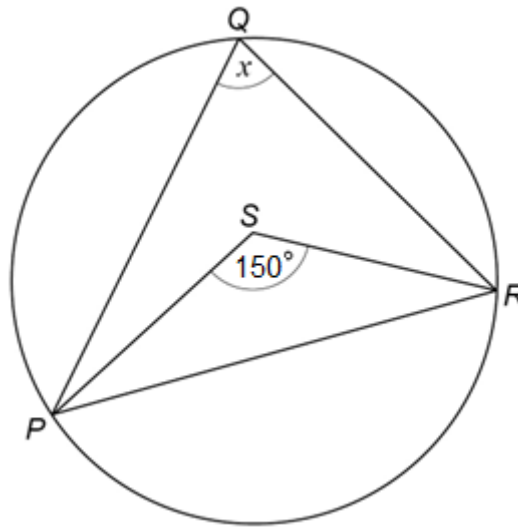
☐

It is smaller than the answer to part (a)

☐

It is impossible to tell

- 20 (a)** P , Q and R are points on a circle.
 S is a point inside triangle PQR .



Not drawn
accurately

Assume that S is the centre of the circle.

Work out the size of angle x .

[1 mark]

$$x = \underline{\hspace{2cm}}^{\circ}$$

- 20 (b)** In fact, S is not the centre of the circle.

What does this mean about the size of angle x ?

Tick **one** box.

[1 mark]

☐

It is the same as the answer to part (a)

☐

It is greater than the answer to part (a)

☐

It is smaller than the answer to part (a)

☐

It could be bigger or smaller than the answer to part (a)

Turn over ►

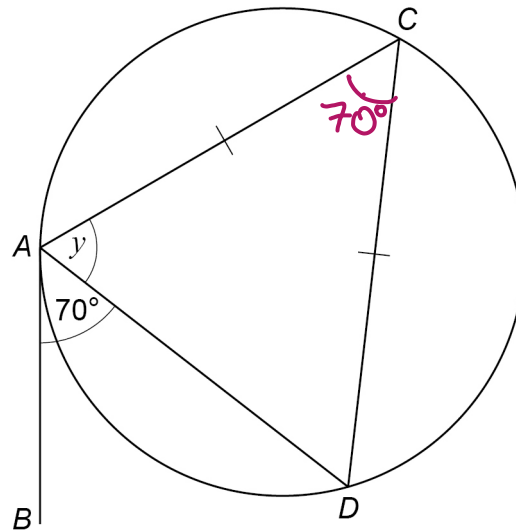
20 (c) For a different circle,

AB is a tangent at A

C and D are on the circumference of the circle

$AC = CD$

Not drawn
accurately



Here is Simon's method to work out the size of angle y .

Angle $ADC = 70^\circ$ (alternate segment theorem)
Therefore $y = 70^\circ$ (angles in an isosceles triangle)

Is he correct?

Give a reason for your answer.

[1 mark]

No, Angle $ADC \neq 70^\circ$, angle $ACD = 70^\circ$

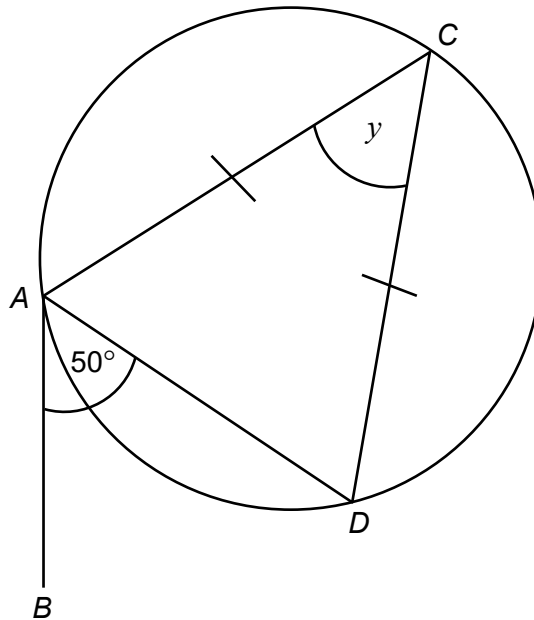
20 (c) For a different circle,

AB is a tangent at A

C and D are on the circumference of the circle

$AC = CD$

Not drawn
accurately



Here is Ollie's method to work out the size of angle y .

Angle $ADC = 50^\circ$ (alternate angles are equal)

Angle $CAD = 50^\circ$ (angles in an isosceles triangle)

Therefore $y = 80^\circ$ (angles in a triangle)

Is he correct?

Give a reason for your answer.

[1 mark]

21

Magana decides to put £500 into an account that pays compound interest.
She wants to have **at least** £560 in the account after 3 years.

Work out to 1 decimal place the **minimum** annual interest rate she needs.

[3 marks]

$$500 \times x^3 > 560$$

$$x^3 > \frac{560}{500}$$

$$x^3 > 1.12$$

$$x > \sqrt[3]{1.12} = 1.03849882 = 103.849882\%$$

Needs 103.9%

Answer 3.9 %

21

Asmae decides to put £2500 into an account that pays compound interest.
She wants to have **at least** £3200 in the account after 5 years.

Work out to 1 decimal place the **minimum** annual interest rate she needs.

[3 marks]

Answer _____ %

Turn over ►

- 22** An approximate value of a root of an equation, x , can be found using the iterative formula

$$x_{n+1} = \sqrt[3]{5(x_n)^2 - 2x_n - 3}$$

The starting value is $x_1 = 4$

- 22 (a)** Work out the values of x_2 and x_3

[2 marks]

$$x_2 = \sqrt[3]{5(x_1)^2 - 2(x_1) - 3} = \sqrt[3]{5(4)^2 - 2(4) - 3}$$

$$x_3 = \sqrt[3]{5(x_2)^2 - 2(x_2) - 3} = \sqrt[3]{5(\text{Ans})^2 - 2(\text{Ans}) - 3}$$

$$x_2 = 4.10156593$$

$$x_3 = 4.177641648$$

- 22 (b)** By continuing the iteration, show that the value of x is more than 4.25

[1 mark]

$$x_4 = 4.234116771$$

$$x_5 = 4.275769447$$

$$x_6 = 4.306345111$$

- 22** An approximate value of a root of an equation, x , can be found using the iterative formula

$$x_{n+1} = \sqrt[3]{7(x_n)^2 - 4x_n - 5}$$

The starting value is $x_1 = 5$

- 22 (a)** Work out the values of x_2 and x_3

[2 marks]

$$x_2 = \underline{\hspace{10cm}}$$

$$x_3 = \underline{\hspace{10cm}}$$

- 22 (b)** By continuing the iteration, show that the value of x is more than 5.85

[1 mark]

23

Here are three sets of cards.

Set A

1	1	3	5	5	5	6	8
---	---	---	---	---	---	---	---

Set B

1	2	4	6	8	8	9
---	---	---	---	---	---	---

Set C

3	4	5	6
---	---	---	---

In a game, a player has two options.

Option 1

Pick two cards from Set A

Option 2

Pick one card from Set B
and
pick one card from Set C

The cards are picked at random.

The player wins if the total of their two cards is exactly 10

Which option gives a better chance of winning?

Option 1

☒

Option 2

☐

Show working to support your answer.

[4 marks]

Option 1 - 2 '5's $\frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$

Option 2 - 4 and 6 $\frac{1}{7} \times \frac{1}{4} = \frac{1}{28}$

or

6 and 4 $\frac{1}{7} \times \frac{1}{4} = \frac{1}{28}$

$= \frac{2}{28} = \frac{1}{14}$

23

Here are three sets of cards.

Set A

1	2	3	3	6	6	6	8	8	8
---	---	---	---	---	---	---	---	---	---

Set B

1	1	2	4	7	7	8	8	10	10
---	---	---	---	---	---	---	---	----	----

Set C

3	3	3	6	6	7	8	8	9
---	---	---	---	---	---	---	---	---

In a game, a player has two options.

Option 1

Pick two cards from Set A

Option 2

Pick one card from Set B
and
pick one card from Set C

The cards are picked at random.

The player wins if the total of their two cards is exactly 12

Which option gives a better chance of winning?

Option 1

Option 2

Show working to support your answer.

[4 marks]

Turn over ►

24

 $a = 45$ to the nearest integer $b = 70$ to 1 significant figureWork out the **upper bound** for $6a^2 - b^2$ You **must** show your working.**[3 marks]**

$$44.5 \leq a < 45.5$$

$$65 \leq b < 75$$

$$\text{upper bound} = 6 \times a_{\max}^2 - b_{\min}^2$$

$$= 6 \times 45.5^2 - 65^2$$

$$= 8196.5$$

Answer 8196.5

24

 $a = 65$ to the nearest integer $b = 30$ to 1 significant figureWork out the **upper bound** for $2a^2 - b^2$ You **must** show your working.**[3 marks]**

Answer _____

25

Show that $\frac{x-7}{x-4} + \frac{x+7}{x+4}$

simplifies to $\frac{ax^2-b}{x^2-16}$ where a and b are integers.

[3 marks]

$$\frac{x-7}{x-4} + \frac{x+7}{x+4}$$

$$= \frac{(x-7)(x+4)}{(x-4)(x+4)} + \frac{(x+7)(x-4)}{(x+4)(x-4)}$$

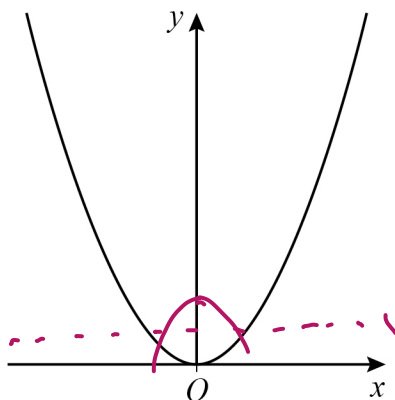
$$= \frac{x^2-3x-28}{x^2-16} + \frac{x^2+3x-28}{x^2-16}$$

$$= \frac{2x^2-56}{x^2-16} \quad a=2 \quad b=56$$

Show that $\frac{x-5}{x-2} + \frac{x+5}{x+2}$

simplifies to $\frac{ax^2-b}{x^2-4}$ where a and b are integers.

26 Here is a sketch of $y = x^2$



26 (a) The minimum point of $y = x^2$ is at $(0, 0)$

Write down the coordinates of the minimum point of $y = x^2 - 3$

[1 mark]

Answer (0 , -3)

26 (b) The graph $y = x^2$ is reflected in the line $y = 1$

Write down the equation of the graph after this transformation.

[1 mark]

Answer $y = 2 - x^2$

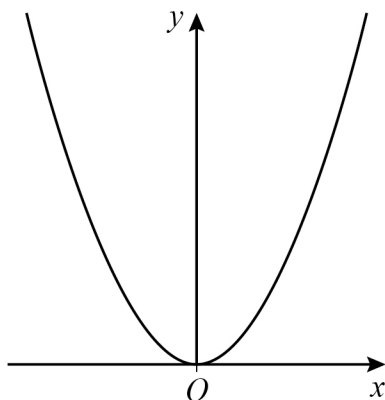
26 (c) $y = x^2$ is now transformed to give $y = (x - 2)^2$

Describe fully this single transformation.

[2 marks]

Translation by the vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

26 Here is a sketch of $y = x^2$



26 (a) The minimum point of $y = x^2$ is at $(0, 0)$

Write down the coordinates of the minimum point of $y = x^2 + 2$

[1 mark]

Answer (_____ , _____)

26 (b) The graph $y = x^2$ is reflected in the x axis.

Write down the equation of the graph after this transformation.

[1 mark]

Answer _____

26 (c) $y = x^2$ is now transformed to give $y = (x + 3)^2$

Describe fully this single transformation.

[2 marks]

END OF QUESTIONS